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What is mathematics?

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Abstract. In this paper I = “James Kuodo Huang” and He = “Paul Erdős”. Once in 1975-77 when I was playing GO game with Professor Erdős, I asked him a question "What is mathematics?". He answered me right away "Mathematics is to solve problems and mathematics is to develop theories “. His view point of mathematics will be the center spirit of this article.

1. Introduction

Ancient Chinese philosopher Lau Zi said “Dao ke dao fei chang da (道可道非常道)” which can be translated into English for people as follows “If a DAO can be described in words, then it cannot be a DAO forever”. Even though I am not really a Lau Zi follower, what he said above is quite true in general, where DAO (道) is a word in Chinese which cannot be translated into any English word exactly. DAO can be an eternal rule, or method or algorithm or theory or standard or other meaning not mentioned. This word could become new word in English. What is a good definition of any field is always very hard. What is the DAO of mathematics? I have had this question since I was young. The closest answer of this question was not answered until I met Paul Erdős. Once in 1975-77 when I was playing GO game with Professor Erdős, I asked him a question "What is mathematics?". He answered me right away "Mathematics is to solve problems and mathematics is to develop theories “. This is very close to the DAO of mathematics. His view point of mathematics will be the center spirit of this article. His theory of mathematics is short and nice.

In order to solve Hilbert second problem and Hilbert sixth problem, James Kuodo Huang discovered Hilbert logic in 2005 ([1], [2], and [3]). Hilbert logic is to be defined as an extension of a consistent Boolean logic so that the universal consistent theorem can be proved. Based on this definition, Hilbert logic theory has been developed by me. The relation of his theory of mathematics and my theory of logic will be addressed in this article.

2. What is mathematics?

In 1975 I met great mathematician Paul Erdős in the University of Florida when I was a first year graduate student in the department of mathematics and Paul Erdős was an invited speaker there. Professor Erdös was not only a great mathematician but also is one of the best mathematicians in the human history. Besides he is a person with great humanity spirit. Professor Erdős and I had a common interest in playing a Chinese game called “GO". Professor R. C. Hwang (He has been a professor in department of mathematics of National Tsing Hua University in Taiwan from 1978-now) was the UF Go club Chair and a Ph.D. student of UF then. Since R.C. and I were two best GO players (about Japan amateur three Dan in 1975) in University of Florida at that time, so R. C. invited me to play a GO game with Paul Erdős

one day. Since then I had played GO game with Professor Erdős many times either in UF or in University of North Texas. Maybe only once was in Professor Mauldin's house in Denton, Texas. By the way my GO level has reached about amateur eight Dan since 2009. One time when I was playing GO game with professor Erdős in UF, I asked him a question "What is mathematics?". He answered me right away "Mathematics is to solve problems and mathematics is to develop theories.". Professor Erdős told me that he was more a problem solving type of mathematician. Now I think myself as more a theory developing type of mathematician and computer scientist. I have quoted his answer of "What is mathematics" to many mathematicians, scientists, and my former students. After 37 years I still think his answer is the best for this question "What is mathematics?". In this article, I will not only tell you why his answer still is the best but also I will tell some of my theory about his answer. First I would like to give you one simple reason why his answer is still the best. His answer works for other fields as well. For instance, let us ask a question "What is Physics?". The answer could be "Solving physics problems and developing physics theories.". His answer actually is his theory of mathematics. Because all people are solving their problems every day and are developing theory related to their problems. From his answer, there are many scientists are also mathematicians too. A theory of mathematics can be developed from his answer.

There are different levels of his answer. I realized his answer for me should be related to unsolved mathematical problems. His answer classified mathematicians into two types, problem solving mathematicians and theory developing mathematicians. Problem solving mathematician is a mature mathematician who would like to use all the known theories, algorithms, methods, strategies, and principles to tackle an unsolved problems. If he succeeds or fails, then he will go to next unsolved problems. A theory developing mathematician likes to develop his own theories to attack an unsolved problem. According to my theory, the problem solver and theory developer are different from their interests too. W. T. Gowers in his paper titled "The two culture of mathematics" in [4] also talked about these topics. Professor Gower wrote "The "two cultures" I wish to discuss will be familiar to all professional mathematicians. Loosely speaking, I mean the distinction between mathematicians who regard their central aim as being to solve problems, and those who are more concerned with building and understanding theories". According to Gower's point of view, I am problem solver as well as theory developer. My purpose to develop theory is trying to solve unsolved problems most of times. For instance, I have developed "Hilbert logic theory, Hilbert-Huang Logic, Hilbert-Huang algebra, and product logic theory" ([5], [6], [7]) because I was trying to solve Hilbert second problem and Hilbert sixth problem. I always like to develop my own theory to solve any problem. Therefore I think of me as a theory developer more than a problem solver. Also I have my own theory about problem solver mathematician. Problem solver type of mathematician has the following characteristics: (1) Mathematics genius (2) mature enough in some fields of mathematics (3) likes to collaborate with others. Theory developer likes to work his own in general. These are not the only my theory on mathematicians. I also have other theory on algebraic mathematician and geometric mathematician.

Why a mathematician become more a problem solver and another mathematician become more a theory developer? I would like to use Professor Erdős and me as two cases to study. According to my theory, Professor Erdős is more problem solver because he possessed the three conditions I mentioned earlier. He liked to collaborate with other mathematicians because he liked

to raise questions and maybe he did not like to write papers. Maybe he liked to help others to understand mathematics. I am more a theory developer and I do not like to write papers as well. When Professor Erdős played "GO" game with me, he did raise unsolved problem to me sometimes. But I never tried his problems. The prime numbers problem related to arbitrarily long arithmetic progressions did inspire me to raise similar question on my own algebraic number theory in 1980s. It also inspires me to study the distribution of quantum algebraic numbers. Professor Erdős liked to play the most sophisticated game "GO" and became quite a good player. But he never tried to develop any theory to play this game, probably because he was a problem solver mathematician and he only enjoyed playing the game for relaxation. To be a good GO player one must learn all the theories and techniques. To be a good professional GO player one must have one's own theory to be successful. I am a theory developer mathematician and I like to develop theory for anything I am interested in. I have also developed my own theory to play "GO" game. Right now I can play any professional GO player an even game and win if I am lucky enough on the game. GO players and Chinese Chess players are two different types of talent. One Grand master Chinese chess player with GO level amateur (Japanese) 6 Dan level cannot play a GO professional player an even game. I have never seen any person who is both GO professional level and Chinese chess pro level. I can give two handy to most master Chinese chess players who I know to play GO game up to now. But they can give me handicap on Chinese chess game. I can play Chinese chess mathematically but I never enjoy the chess game as much as I enjoy the GO game. I had no talent to play Chinese chess games because I can replay a GO game that I just finished the game but I cannot do the same with chess game. Mathematician with chess talent will be different from mathematician with GO game talent.

Let me give you two examples to show you how Professor Erdős likes to play GO game. In 1976-77 one evening in University of Florida, I had my dinner and went to my graduate student office and worked. The phone was ringing. Edward Horworka picked up the phone and told me Professor Erdős was looking for me. He wanted to play me a GO game in the department faculty meeting room. Then I went there to play the GO game. In 1977-80, one time, I attended his invited talk in UNT and sit in end of the classroom. Before the end of the talk, he was talking and walking towards my seat and asked me if I had time to play a GO game after the talk. Of course I said yes. Afterwards he finished the talk and let people asked him questions before we started the GO game. I even developed some theory about "NP VS P" problem 1997.

I would like to investigate myself more why I would become a more theory developer. Let me tell you my experience with mathematics. I graduated from one of the best high school in Taiwan, Taipei Jian Guo High school. Because competitive joint entrance examination to universities, I had never had a chance to know any mathematics higher than calculus level in high school. When I was a freshman as a math major in National Taiwan Normal University, I began to read a lot of math books by myself because I found almost all the mathematics textbooks used in the required mathematics courses were too low level. I can finish naive set theory in one day but I found set theory and topology books by Bourbaki more challenge. All the mathematics books which I purchased in Taipei Book store before 1970 were graduate textbook in English version. The way I read any math subject, I would like to prove a theorem by myself first more than to read the proof in the book. I would like rather to skip the proof if I can understand the subject without going through the proof. Many times I had to prove the theorem before I could really understand

the topics. I had never tried any unsolved problem when I was undergraduate student. The first time I encounter an unsolved problem was in the University of Florida, in 1975. There are a lot of good thing happened to me mathematically there (September, 1975 to June 1977). I would like to list them as follows:

(1) I met professor Paul Erdős when he was 63 years old and he inspired me a lot. I think I still have time to do a lot of mathematics from now on. From 1980 to 2005 I did more computer science and computer technology. But I still worked on famous unsolved problems off and on during this period of time. From 2009 to now, I devoted to mathematics research again.

(2) I met Professor Kazimierz Kuratowski in UF and ask him one question after his invited talk, "How to be a famous mathematician?". His answer was "If you live old enough".

I still like his answer because I am very healthy and have chance to live old enough.

(3) I met Professor Edward Horworka and he told me that to solve a Hilbert unsolved 23 problems could make one famous. He also told me to order 1976 proceedings on Hilbert 23 problems from American Mathematical Society. He was also a graduate student in UF then. He becomes my life time friend.

(4) I met Professor Neil White. One time I asked him "Why I found so hard to solve some unsolved problems?". His answer was "Don't think anybody can.". That is a very good attitude.

(5) I met Professor R. Daniel Mauldin in UF. He became my life time academic friend.
We have one thing in common. We both like all kinds of mathematics.

(6) I met Stanley Ulam in UF. We had two common interests. One is mathematics and the other one is playing table pool game. He even invented his own Ulam Billiard game on 9 ball table which was very similar to the professional Taiwanese 9 ball players practice game. When he found out I was one of the best pool player in University of North Texas 1980, we did play his game once in UNT.

(7) From 1975-77, the professors and graduate students were all very nice to me in University of Florida. I wish to go back there as a visiting professor for one quarter someday.

I should thank Edward Horworka for letting me encounter Hilbert 23 problems in 1976. I began to read the Hilbert problems in the proceedings that I purchased. I finally realize Bourbaki's work were Hilbert School in France. Almost all fields of mathematics are influenced by Hilbert School more or less. The Bourbaki group has done a lot for the developing mathematics education

in more strictly discipline manner. I actually enjoy more naïve introduction of any mathematics. These two approaches to study mathematics are both important.

The general problem solving approach between problem solver and theory developer are a little different. For instance, if both of them use "divide and conquer" method to tackle a math problem. The problem solver can quickly reach the solution if a question has solution by using known knowledge and technology. The theory developer would go into developing a new theory if he find an interesting theory in the middle of his divide and conquer process. The theory developer can solve the problem only if his new theory can reach the solution of the problem too. In general, the problem solver will solve the problem first. For a long standing unsolved problem, a good theory developer probably will solve the problem.

What is science? The best answer probably is still "Solving scientific problems and developing theory of science". If one day we would like to unify all sciences into one big field this could be the choice of definition. The advantage of this type of answer is simple and beautiful but the disadvantage of this type of answer is it does not give too much of the inside of any field.

Is there a field which has a definition with the spirit of Professor Erdős's answer for what is mathematics and also give much inside of field? The answer is affirmative. The definition of the field is "Hilbert Logic Theory". In order to solve Hilbert second problem and Hilbert sixth problem I have developed Hilbert logic theory in [1], [2], [3], [5], [6] and [7]. Hilbert logic is to be defined as an extension of a consistent Boolean logic so that the universal consistent theorem can be proved. This definition of Hilbert logic theory has the spirit of Professor Erdős's answer for what is mathematics. The reason is as follows. This definition also implies sound and consistent property of Hilbert logic. The Hilbert logic theory can make model theory, proof theory, many logic theories and many computer science logic theories as subfields if we develop the Hilbert logic in very careful manner.

In the end I would like to point out one thing, deep inside Professor Erdős' s heart, he believes in God because he liked to help others. One day he told me that if he died the first question he would like to ask God was "Whether the continuum hypothesis is right". I saw through his eyes and realized that he did believe in GOD and he really loved mathematics. He liked not only Jewish people and Israel but also He liked other people as well. If mathematics is also a game, then there were certainly two games he liked before another game was GO. If anyone who wrote story about Paul Erdős without mentioning about GO, he must miss a lot things.

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